Makeup Exam for Problem Session (UE)

25.2.2021

Problem 1. A sequence $(a_n)_{n\geq 0}$ in $\mathbb{R}$ satisfies

$$-2a_n + 3a_{n-1} + 2a_{n-2} = 0 \quad (n \geq 2), \qquad a_0 = \tfrac{11}{2}, \qquad a_1 = \tfrac{49}{4}.$$

Determine the asymptotic growth of $(a_n)_{n\geq 0}$.

Problem 2. We denote by $\mathbb{C}^k$ the space of k-dimensional column vectors over $\mathbb{C}$, and by $\mathbb{C}^{k\times n}$ the space of $k \times n$-matrices. Show that

$$\mathbb{C}^k \times \mathbb{C}^n \to \mathbb{C}^{k\times n}, \qquad (x, y) \mapsto xy^T$$

is a tensor map.

Problem 3. Let $G = (V, E)$ be a graph and $d_G(v) = 1$ for some $v \in V$. Show that G is a tree if and only if $G[V \setminus v]$ is a tree.

Problem 4. Let R be a ring and let $f : M \to N$ be an R-epimorphism. Show: if N and $\ker(f)$ are finitely generated, then M is finitely generated.